Identification of Quasi-Normal Modes

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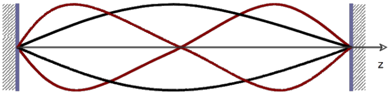
Agenda

- Introduction
    - Normal Modes
    - Open Systems
    - QNMs
- Objective of the thesis
- Experiments and Results
- Conclusion
- Future Work

Introduction

- Certain systems when excited by an external input dissipate the stored energy in the form of vibrations at its natural frequencies constrained by its geometry and composition
  - Guitar String
  - Optical resonator: Light trapped using reflecting boundaries



Introduction

- Simulating a system governed by partial differential equations is computationally intensive
- Hardware is limited for the size of systems that need to be simulated and solved
- Quasi-Normal Modes approach requires much less computation than solving the partial differential equations
- The presentation is towards understanding and identifying the QNMs for such systems

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Normal Modes

Normal Modes

- Systems such as guitar strings releases energy as vibrations only at one frequency in a pattern called the mode of the system at that frequency
- Electric field in an optical cavity when excited by inputs of different frequencies
- When input contains more of these frequencies, output is the sum of modes at those frequencies

- Vibrations of different frequencies undergo different level of damping
- The frequencies for which the damping is small are called natural frequencies
- The phenomena is called resonance

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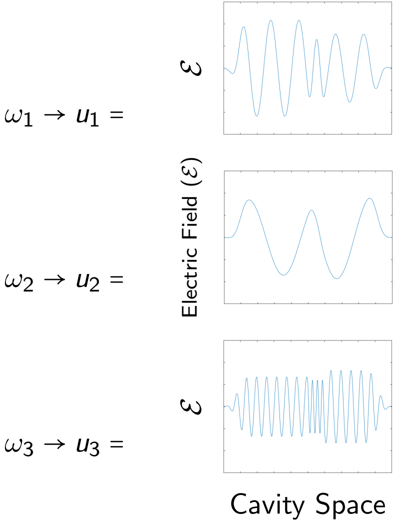

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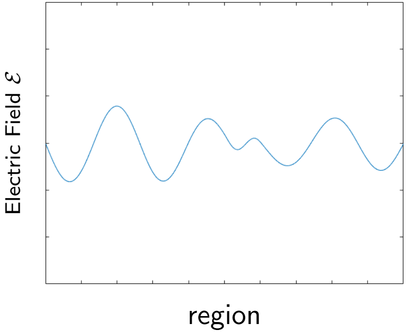

Normal Modes and Natural Frequencies

- It is the motion of the field inside the system at its natural frequencies
- The associated natural frequencies are real
- Completely describes the behavior of a system (Modes are complete)
- For inputs containing multiple natural frequency, output is the sum of modes at that frequencies
- This means that some modes are more dominant than others in determining the solution
- Reduces the computation requirements

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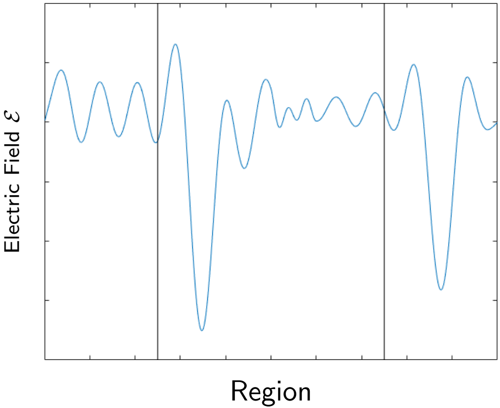
Open System

- System loses energy to infinity
- Damping can't be accurately modeled using the normal modes



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Quasi Normal Modes

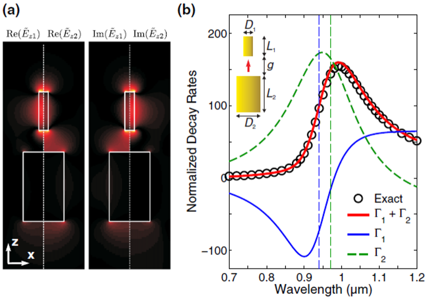
- The modes of such open system are called Quasi-Normal Modes
- They have complex eigenvalues with imaginary part being the damping in the system
- Doesn't fit the definition of Normal modes because of the complex frequencies
- Its ability to completely describe a system in terms of QNM expansion has been observed experimentally

QNM Completeness

Gold Nano-Rod Experiment

Theory of spontaneous optical emission by C. Sauvan

- The decay rate of two gold nano-rods was modeled using the sum of responses of the individual gold nano-rods in terms of their QNMs
- QNMs are a powerful tool in representing a system governed by small number of resonant modes
- A discretized system has several modes
- It is a difficult task to identify the QNMs among those modes



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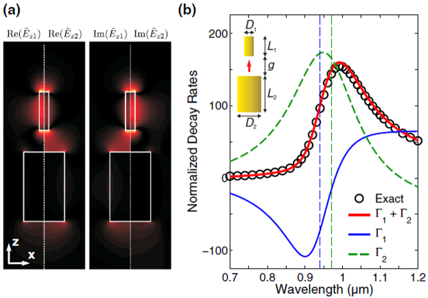

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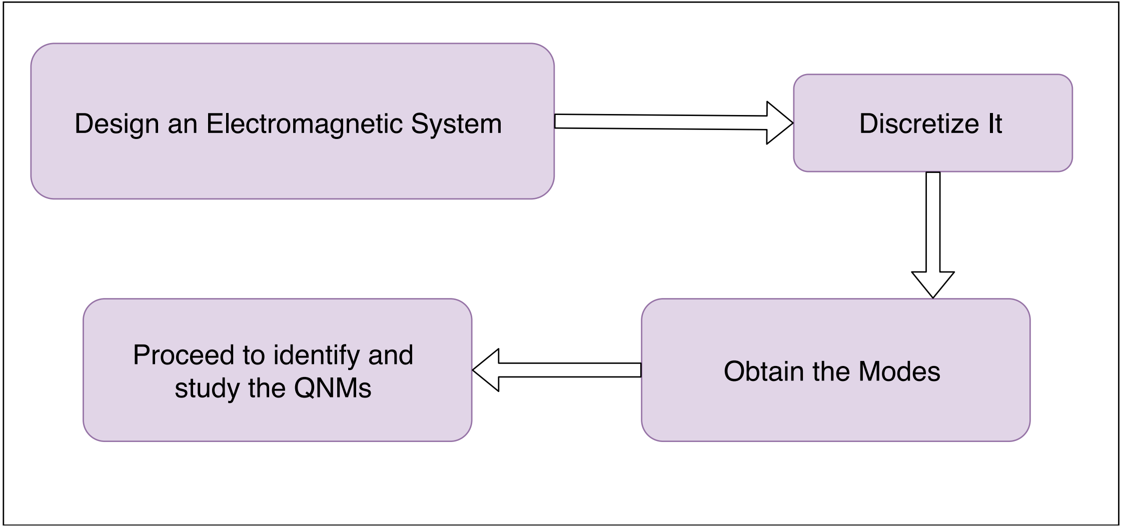
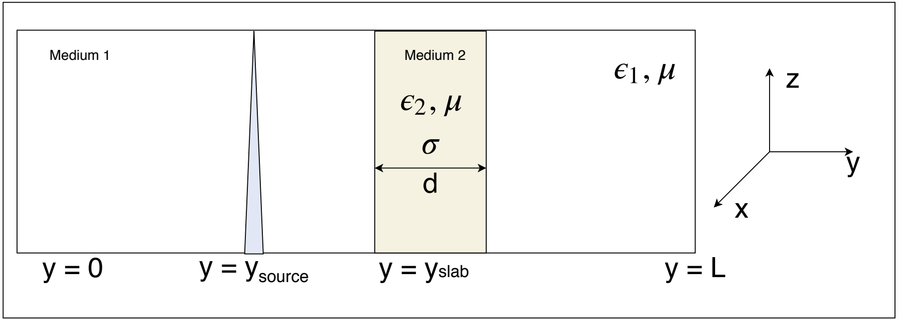
Objective

- To obtain the Modes of a chosen system
- To obtain a structure of the modes and identify the QNMs
- To construct the solution using the Quasi-Normal Modes containing the input frequency

Implementation



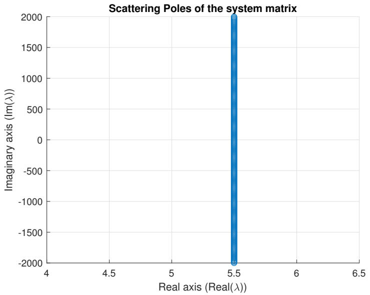
Electromagnetic System



- A one-dimensional EM system with a slab and a source is considered
- Loss in the system is represented by electrical conductivity σ
- Medium parameters are the electrical permittivity ϵ_2, magnetic permeability μ and the slab width d

Electromagnetic System

- An analytical solution is available in the literature
- Scattering Poles can be obtained

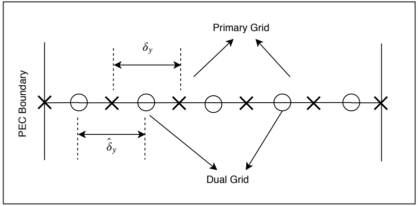


Discretization

One dimensional Maxwell equation governing the EM waves inside the system are:

Maxwell Equations

$$\partial_y \mathcal{H}_x + \sigma \mathcal{E}_z + \epsilon_r \partial_t \mathcal{E}_z = -\mathcal{J}_z^{ext}$$
$$\partial_y \mathcal{E}_z + \mu_r \partial_t \mathcal{H}_x = -\mathcal{K}_x^{ext}$$



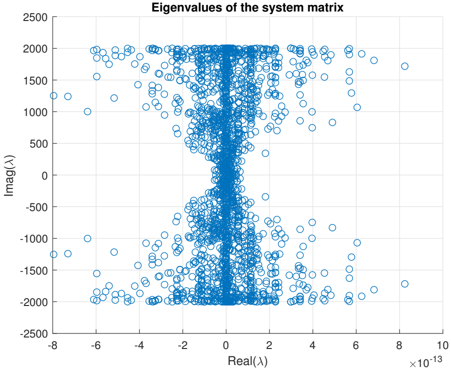
- Finite-Difference approach is used
- The system is discretized on a grid like structure

System Equation

Discretized model of the system is $(A + sI)\mathbf{f} = -\mathbf{q}$, $A = M^{-1}(D + S)$
Solution $\mathbf{f}(t) = Ve^{-\lambda t}\mathbf{c}$

Matrix A for a lossless system ($\sigma = 0$)

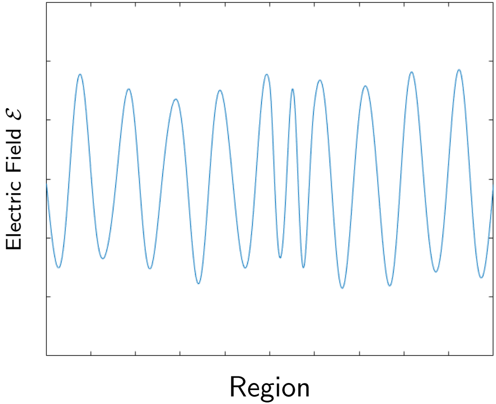
- Eigenvalues of the matrix A are purely imaginary
- From our definition of the system equations, it shows a purely oscillatory behavior with no damping.
- Waves oscillate forever without decaying



Reflections

Truncation with PEC conditions creates a boundary that causes reflections.

- Discretized system not identical to the open system
- Outgoing waves getting reflected instead of leaking to infinity
- An absorbing layer called a Perfectly Matched Layer is implemented
- A PML simulates the extension to infinity.



PML

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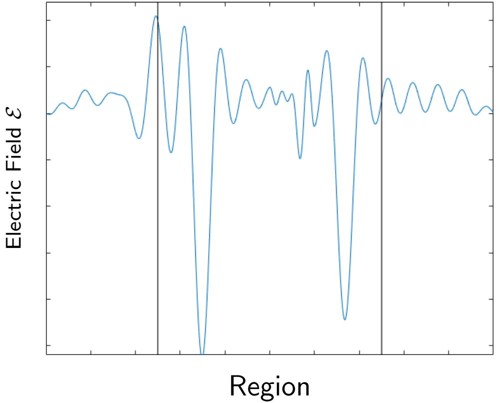


Modified System Equation

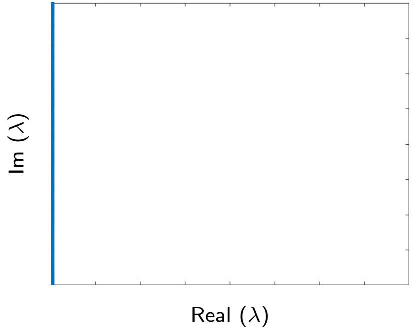
Discretized System with PML Implemented

It can be shown that even after the addition of PML, the system equation can still be expressed as:

$$(A + sI)f = -q,$$

where $A = M^{-1}(D + P + S)$

- Eigenvalues of matrix A shifts to a region of higher damping



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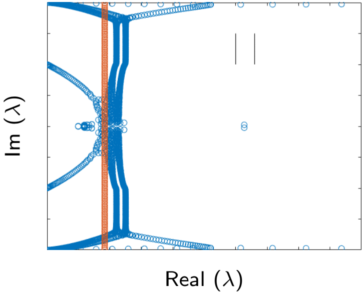
Modes

- New modes were introduced because of discretization and introduction of PML
- We observe the eigenvalue distribution and its eigenvectors as the system parameters are changed

- Slab Width
- PML Length
- Loss σ in the slab

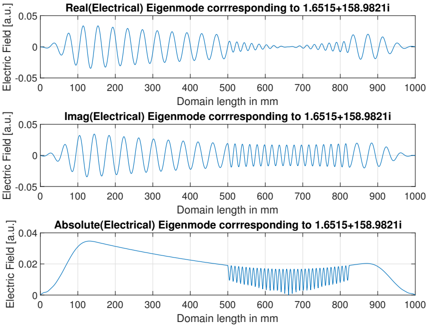
Slab Width

- The red stripe is the scattering poles calculated from the analytical model
- There are multiple stripes
- The left stripe overlapping with the scattering poles are the QNMs
- The slab width is increased by shifting the right end of the slab
- On increasing the slab width, the middle stripe does not change but the other two move apart
- We observe the eigenvectors of the different stripes
  1. Left Stripe
  2. Middle Stripe
  3. Right Stripe



Slab Width

- The red stripe is the scattering poles calculated from the analytical model
- There are three distinct stripes
- On increasing the slab width, the middle stripe does not change but the other two move apart
- We observe the eigenvectors of the different stripes
  1. Left Stripe - Eigenvectors are spread over the entire region originating from the slab
  2. Middle Stripe
  3. Right Stripe



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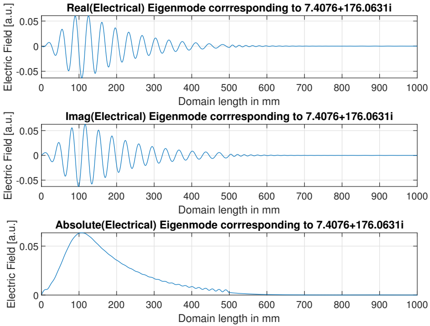
  1. Left Stripe
  2. Middle Stripe - It shows wave propagating in the left direction originating from the slab
  3. Right Stripe



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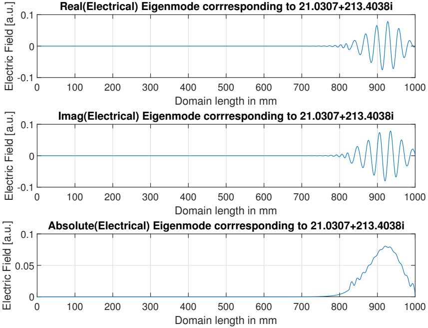
- We observe the eigenvectors of the different stripes
  1. Left Stripe
  2. Middle Stripe
  3. Right Stripe - It shows wave propagating in the right direction originating from the right end of the slab



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- Slab Width
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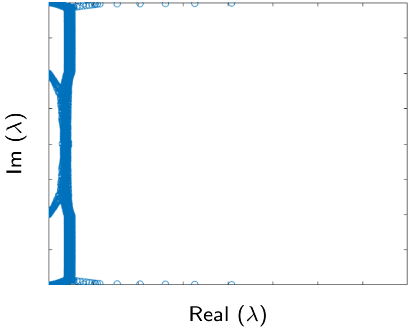
PML Length

- As PML Length is increased, more eigenvalues separate in the region of horizontal stripe
- A greater part of the system falls inside the damping region of PML
- The eigenvector corresponding to the eigenvalue in the horizontal stripe is localized in the PML area



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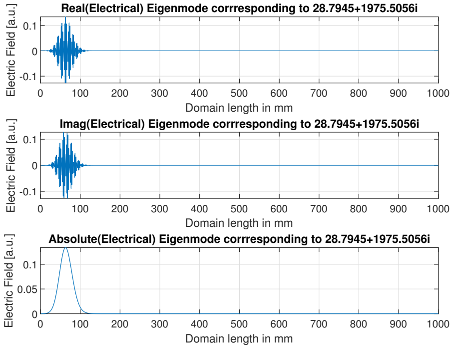
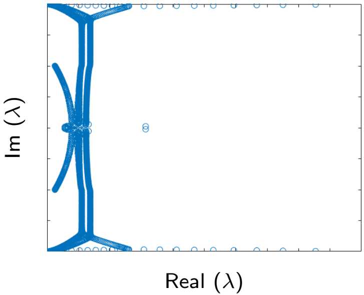


Modes due to lossy medium

- PML is a lossy layer by design which attenuates the waves in a small region with minimal reflections
- In this case, a lossy slab should have a similar behavior
- Therefore, we begin with add more loss to the slab and observe the behavior

Loss σ in the slab

- Loss in the slab is increased
- New eigenvalues start separating into the horizontal stripe
- The corresponding eigenvalues are localized in the slab area



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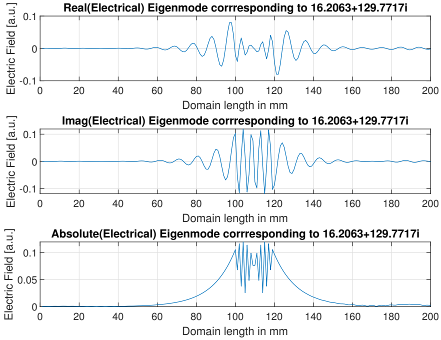
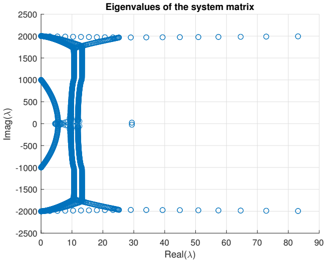


Conclusions

With these results, we have:

- Clustered the eigenvalues
- Established that PML modes behave similar to lossy modes
- Found a structure of the lossy modes
- Identified the QNMs



Reconstructing the solution

We will reconstruct the wave-field solution using the identified QNMs. We take different regions of the QNMs for this purpose. The three regions are:

- Both end of the spectrum excluding the PML modes
- Eigenvalues with the positive imaginary part
- Eigenvalues with the negative imaginary part
- Entire spectrum of eigenvalues excluding the region around the frequency of the input pulse

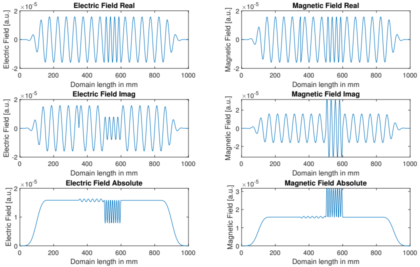


Figure: Finite Difference solution for an input of a chosen frequency

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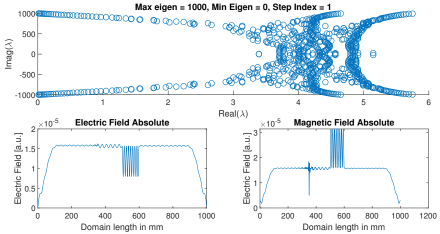

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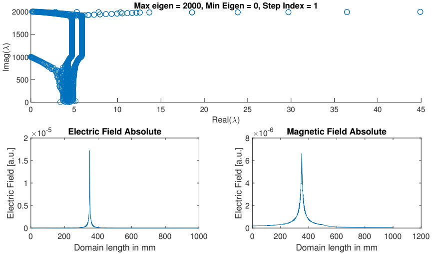

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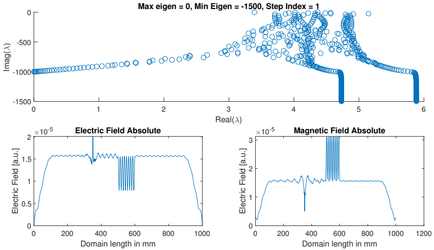

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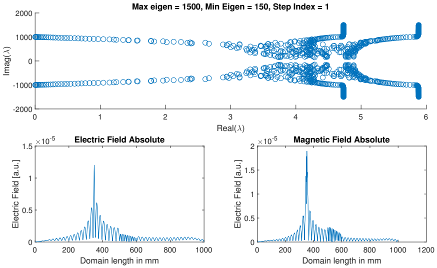

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Conclusions

- Eigenvalue spectrum was clustered into different regions of QNMs and lossy modes
- Lossy regions come out as a separate region in the eigenvalue spectrum if the loss is high compared to other regions
- The separated region is sparsely populated
- High loss regions have their corresponding eigenvectors localized instead of being spread out in the entire region
- QNMs corresponding to the eigenfrequencies lying in the bandwidth of the input pulse is necessary for reconstructing the solution

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Future work

- Using the structure of the QNMs and PMLs to find the relevant modes
- Creating a filter to have a faster convergence to the QNMs
- Using a different system with fewer number of scattering poles

Thank You!