

DELFT UNIVERSITY OF TECHNOLOGY

SC42035

INTEGRATION PROJECT SYSTEMS AND CONTROL

**Project on
Magnetic Manipulation**

Authors

Group 48

Pranav Prakash (4709373)

Satyajith Jujavarapu (4708547)

June 22, 2018

Contents

1	Introduction	2
2	Development of a dynamic mathematical model	2
3	Estimating unknown parameters	2
4	Linearizing the system using input output (I/O) feedback	5
5	State Feedback Control (pole placement)	6
6	Model reference adaptive control by full-state measurement	7
6.1	Design of the reference model	7
6.2	Simulation results	8
6.3	Experimental results	9
7	Gain scheduling controller	10
7.1	Linearization of the system dynamics around an operating point	10
7.2	Controller design for the linearized plants and controller scheduling	11
7.3	Simulation results for gain scheduling	11
8	Results	14
8.1	Pole Placement	14
8.2	MRAC by full-state measurement	14
8.3	Gain Scheduling	15
9	Conclusions	15

1 Introduction

Magnetic manipulation is a technique used to position metallic objects (like a ball in this case) using an array of electromagnetic coils. For this project, the one-dimensional case is considered and the goal is to model the dynamics of the system, estimate all the unknown system parameters and implement relevant controllers to position/regulate the metallic ball quickly, accurately and without overshoot. The given system has four coils with control inputs u_1, u_2, u_3 and u_4 respectively, which are coil currents in the range 0 A to 0.6 A. The position of the ball can be measured using the provided sensor.

2 Development of a dynamic mathematical model

The range over which the ball can traverse along the given track is from 0 mm to 125 mm. The centers of the four coils (numbered 1,2,3 and 4) are at 0.025, 0.050, 0.075 and 0.100 m respectively. One possible analytical approximation for the force on the ball due to the j^{th} coil for the ball position x and the current through the coil i_j is given as

$$F_j(x, i) = g_j(x) i_j^2 = \frac{-\alpha(x - 0.025 \times j)}{((x - 0.025 \times j)^2 + \beta)^3} i_j^2 \quad (2.1)$$

We do not take sliding friction into account since the ball rolls, and although including the rolling dynamics in the mathematical model makes the system description more accurate, it greatly increases the complexity of the problem and hence in determining the controller. Though, fluid friction is taken into account as the medium through which the ball moves can significantly impact the system characteristics. The acceleration of the ball is given by the expression

$$\ddot{x} = -\frac{b}{m}\dot{x} + \sum_{j=1}^4 \frac{g_j(x)}{m} i_j^2 \quad (2.2)$$

where b is the viscous friction coefficient and $g_j(x) = \frac{-\alpha(x-0.025 \times j)}{((x-0.025 \times j)^2 + \beta)^3}$. The dynamics are evidently non-linear and can be represented in the standard form as

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ -\frac{b}{m}\dot{x} + \sum_{j=1}^4 \frac{g_j(x)}{m} i_j^2 \end{bmatrix} \quad (2.3)$$

and in the state-dependent coefficient form as

$$\dot{X} = A \dot{X} + B(X) U \quad (2.4)$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{g_1(x)}{m} & \frac{g_2(x)}{m} & \frac{g_3(x)}{m} & \frac{g_4(x)}{m} \end{bmatrix} \begin{bmatrix} i_1^2 \\ i_2^2 \\ i_3^2 \\ i_4^2 \end{bmatrix}$$

3 Estimating unknown parameters

The parameters α , β and the viscous friction coefficient b need to be determined. While an estimate of α and β were found using a force measurement device, the viscous coefficient b required estimation through looking at damping coefficient in an exponential decaying graph.

Also, since the rolling friction was ignored due to slow response of simulink, estimated value of b did not fit the ball trajectory as well as it should have with the old estimate of α and β .

Therefore another estimate was found for α , β and b through non-linear curve fitting with the trajectory of the ball to a given input with previously estimated values as the initial condition.

This estimation was also verified using Model Reference Adaptive Controller(MRAC)

Initial estimate of α and β :

An initial estimate of the parameters α and β was measured under the assumption that all 4 solenoids share a common value for the parameters α and β . This initial estimation was achieved by measuring force exerted by a single solenoids at different locations for a given value of the current. The equation 2.1 when fitted against the measured values produced the following estimate of α and β

$$\alpha = 2.493 \times 10^{-9} , \beta = 2.66 \times 10^{-4}$$

The force equation for this parameters and the corresponding measured values is shown in figure 1.

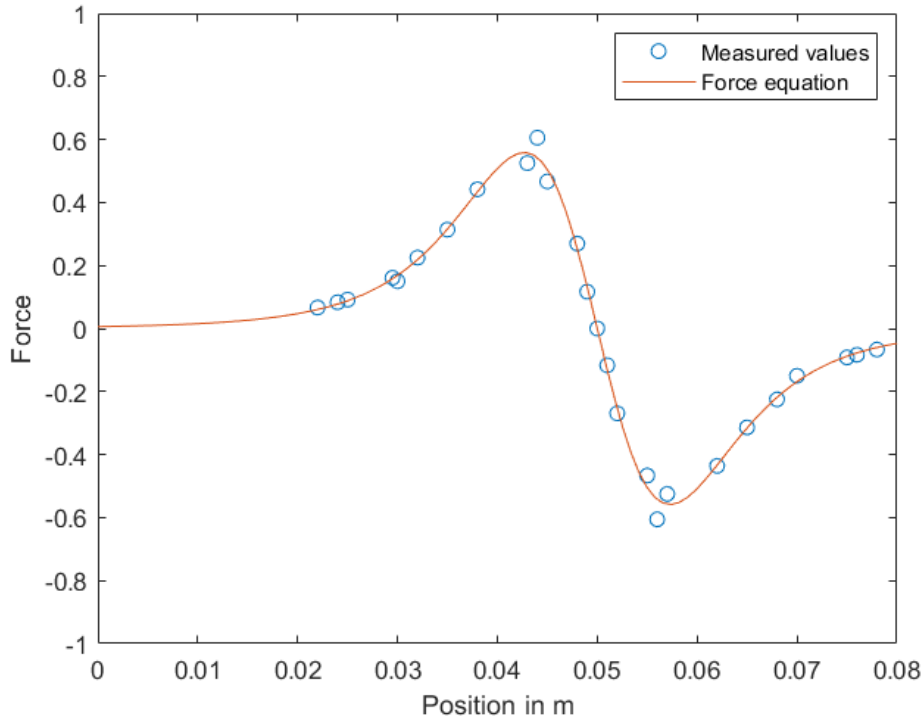


Figure 1: Force equation fitted with the measured values of force exerted by a single coil at different positions

Estimation of viscous coefficient b through exponential fitting

Under the effect of force due to a constant current through a single coil, the ball oscillates around the coil center due to the presence of an attracting force. Viscous damping in these kinds of system induces an exponential decay in the velocity with time. This decay increases exponentially with the damping coefficient. A general solution for a damped oscillator can be written as :

$$x(t) = e^{-b\omega_0 t} \cos(\omega_d t) \tag{3.1}$$

An estimate of b was found by fitting the peaks of the trajectory corresponding to current through a single coil to an exponential curve as shown in figure 2.

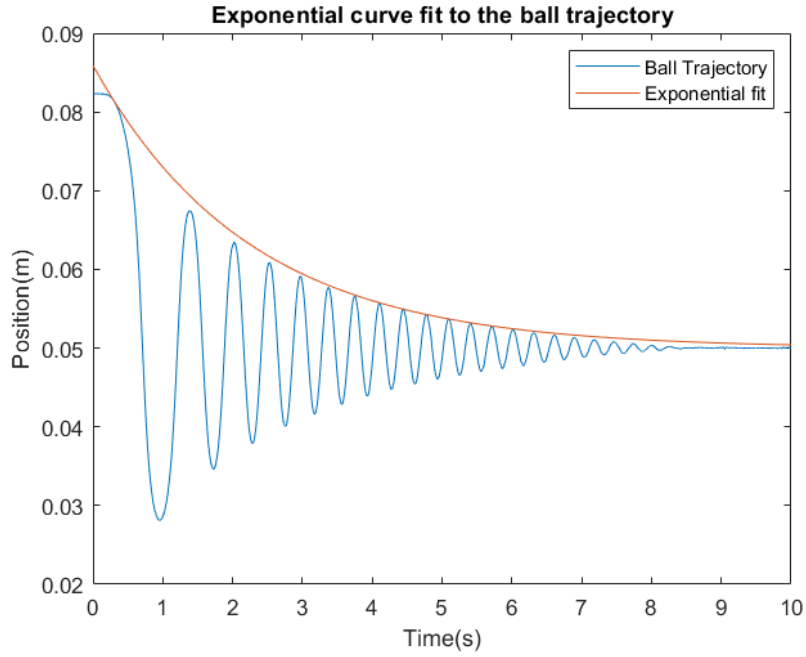


Figure 2: Exponential fit to the ball trajectory

The method employed for the curve fitting is explained below:

- Part of the trajectory below the mean position was ignored and rest of the data was linearized by taking log of the position information as shown in figure 3. Linearized data is obtained for the peaks because the decay is exponential and log of an exponential function is linear. The linearized data obtained is:

$$y = -b\omega_o t \times \log(\cos(\omega_d t) > 0)$$

$\cos(\omega_d t) > 0$ because the position data below the mean position was ignored.

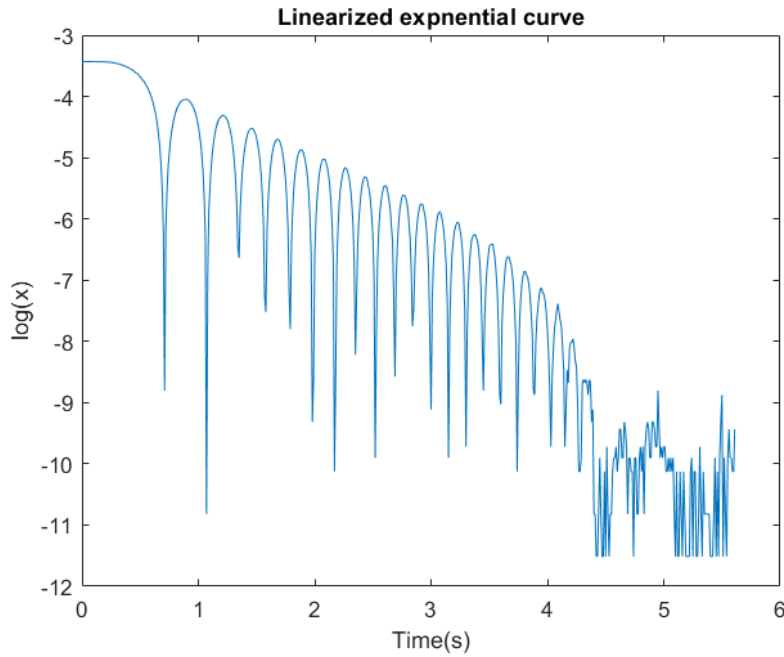


Figure 3: Linearized position curve

- Peaks of y which is linear was fit to a linear function to obtain $C_1 = -b\omega_o t$ and C_2 = the slope formed by the peaks of $\log(\cos(\omega_d t) > 0)$

- From the equation 3.1, ω_d is the frequency of the oscillatory behavior and can therefore be found by measuring the time between consecutive peaks and using this b can be calculated using the following set of equations:

$$C_1 = -b\omega_o t$$

$$\omega_d = \omega_o \sqrt{1 - b^2}$$

- The process was repeated several times and an average value was recorded

The resulting viscous coefficient obtained from this system was

$$b = 0.0361$$

Estimation of α , β and viscous coefficient b through curve fitting

The initial estimate was then used in an estimation where the non-linear simulink model was used to fit the model in the expression 2.3 with the parameters as α , β and b against trajectories obtained from the physical magman model.

The trajectory was obtained by setting a constant current in one of the coil which led the ball to in a decaying oscillating trajectory around the coil. However, this trajectory can't be directly compared the the model assumed here because of the absence of rolling friction in the assumed model. This friction was ignored because of complexity and slow performance of simulink in the physical set-up provided for this experiment. Therefore, a part of the trajectory where it decays to zero is ignored and the rest is used for fitting the model in expression 2.3 as shown in figure 4 (left) with the measured α and β as the starting point. The obtained vales for the parameters were:

$$b = 0.022, \alpha = 1.34 \times 10^{-9}, \beta = 1.91 \times 10^{-4}$$

Apart from this, model reference adaptive control by full-state measurement was used to find an estimate for α and b in real-time with β fixed as the value from the previous estimate. The approach to this method is explained in the MRAC control design section. The results obtained had an oscillation around the previous estimates as show in Figure 4.

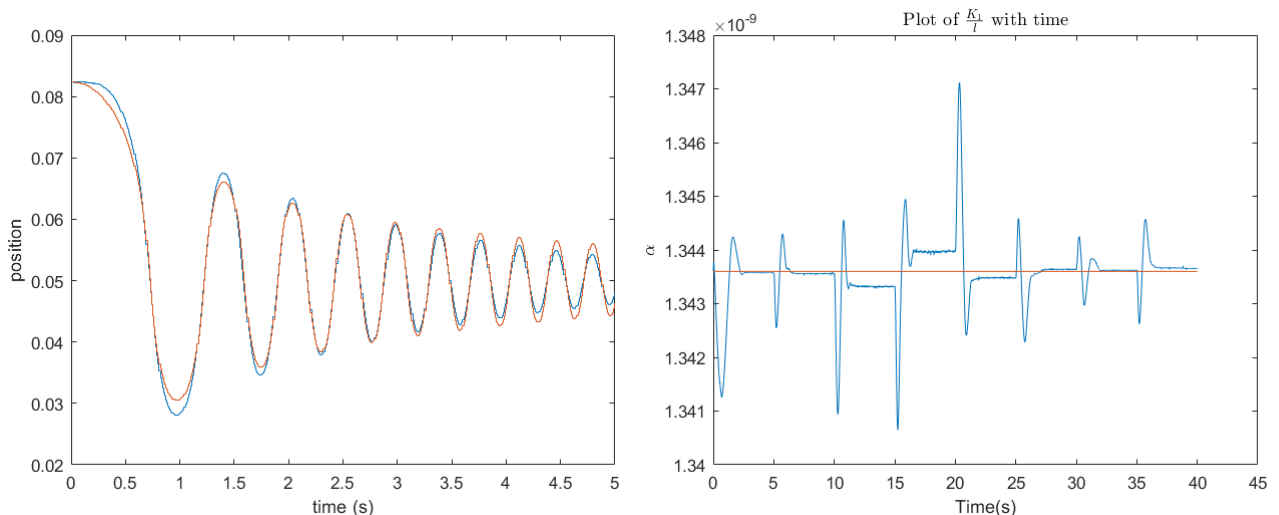


Figure 4: Estimation of α , β and b (left) and estimation of α in MRAC.

4 Linearizing the system using input output (I/O) feedback

I/O feedback can be used to obtain a linear system description from the non-linear system. There is more than one way to do this. One approach is to use the following input

$$i_j^2 = \frac{m}{4g_j(x)} \times \left(\frac{b}{m} \dot{x} + v \right) \quad (4.1)$$

The non linear model reduces to:

$$\ddot{x} = -\frac{b}{m} \dot{x} + \sum_{j=1}^4 \frac{g_j(x)}{m} \frac{m}{4g_j(x)} \left(\frac{b}{m} \dot{x} + v \right)$$

$$\implies \ddot{x} = v$$

The system is reduced to a linear model and linear control strategies can now be applied. The system description for the above linearization is

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \quad (4.2)$$

Another approach is to linearize the system using the following input

$$i_j^2 = \frac{m\alpha}{4g_j(x)} v \quad (4.3)$$

The system description for this linearization approach is

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b}{m} \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha \end{bmatrix} v$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \quad (4.4)$$

Both linearization approaches have been attempted. The linearization input from Equation 4.1 has been used for a simple pole-placement proportional derivative (PD) control scheme. The input from Equation 4.3 can be used to derive the system description presented in Equation 4.4, in which we assume that the state matrices A and B are unknown and can use MRAC by full-state measurement (which is presented in Section 6) to estimate α and b in real-time.

5 State Feedback Control (pole placement)

One of the control implemented was the feedback control using

$$v = - \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + K_r r$$

$$\implies v = -K_1 x - K_2 \dot{x} + K_r r$$

To determine K_r , it is seen that in a general linear state space model, steady state output is given by:

$$X_s = -(A - BK)^{-1} BK_r r, \quad y_s = Cx_s$$

In case of a double integrator model given in the equation 4.2, $A = 0$ and therefore $x_s = \frac{1}{K_1} K_r r$, $\dot{x}_s = 0$ and $y = x_s$. In order for the output to track the reference signal r , it can be easily seen that:

$$K_r = K_1$$

For the discretization of the model, matlab function $sysd = c2d(sys, Ts)$ was used and the poles were placed at $e^{-K_1 T_s}$ and $e^{-K_2 T_s}$.

For a staircase reference signal, the controller tracking is shown in the figure

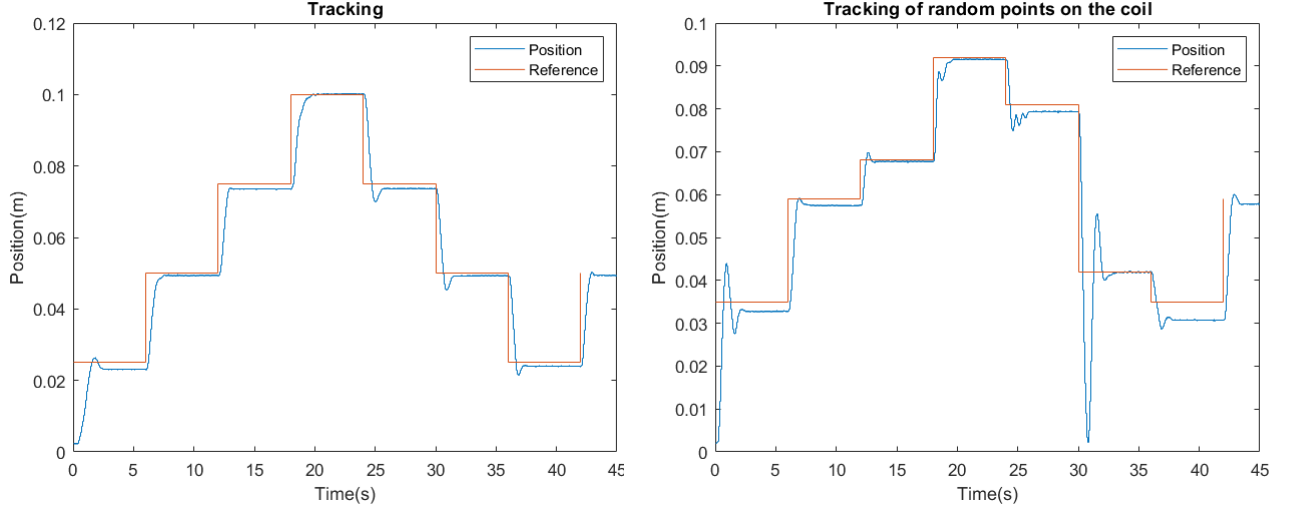


Figure 5: Reference tracking by a PD controller for coil centers and non-center positions.

6 Model reference adaptive control by full-state measurement

In MRAC by full-state measurement, we assume that the plant state matrices A_p and B_p are unknown and devise an adaptive control scheme such that the error between the plant states and the states of a designed reference model is zero. The linear plant dynamics for the MRAC control scheme are expressed in Equation 4.4.

6.1 Design of the reference model

For the design, a reference model was chosen which correctly tracks a desired trajectory. Since the reference signal gives the location of the ball at any time, it is required that state x of the model converges to the reference signal r for some input. The following reference model is chosen:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v \\ y &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \end{aligned} \quad (6.1)$$

Setting $v = -K_1^* x - K_2^* \dot{x} + K_1^* r$, one gets $x = r$ and $\dot{x} = 0$ as the steady state value. Therefore, it can be seen that the model can properly track the reference signal. The feedback linearized reference model is given as:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \left([-K_1^* \quad -K_2^*] \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + K_1^* r \right) \\ \Rightarrow \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -K_1^* & -K_2^* \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ K_1^* \end{bmatrix} r \end{aligned} \quad (6.2)$$

The linearized plant model given in expression 4.4 is controlled by dynamically placing the poles, which reduces the error between reference model states and plant states using linear state feedback

$$v = - [K_1(t) \quad K_2(t)] \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + l(t)r$$

Acceleration of the ball with this feedback law is given by using this input in the expression ?? as given below:

$$\begin{aligned}\ddot{x} &= -\frac{b}{m}\dot{x} + \alpha(-K_1(t)x - K_2(t)\dot{x} + l(t)r) \\ \ddot{x} &= -\alpha K_1(t)x + \left(-\frac{b}{m} - \alpha K_2(t)\right)\dot{x} + \alpha l(t)r\end{aligned}\quad (6.3)$$

The ideal constants which results in the minimization of error between the states is obtained by comparing expression 6.2 and expression 6.3 as given below

$$l = \frac{K_1^*}{\alpha}$$

$$-\frac{b}{m} - \alpha K_2(t) = K_2^*$$

Now, there are two equations in two variables α and b which can be easily solved giving another estimate of these parameters as given below.

$$\alpha = \frac{K_1^*}{l}$$

$$b = m(-K_2^* - K_2(t)\frac{K_1^*}{l})$$

Estimation of α is given in the figure 4.

6.2 Simulation results

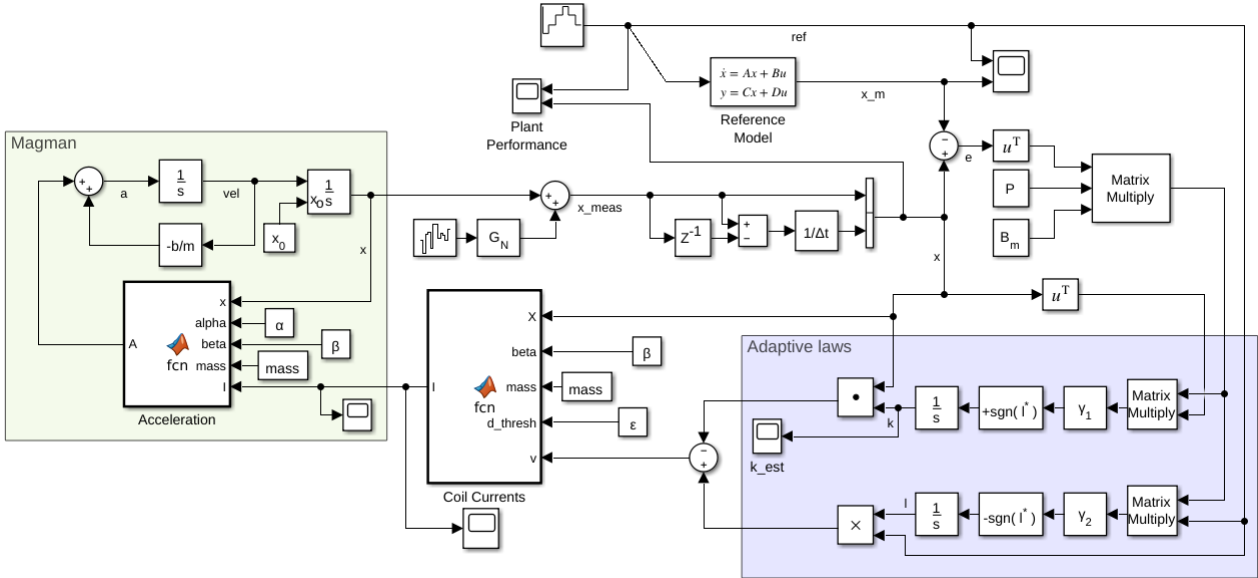


Figure 6: Simulation model used for MRAC

The simulation results show the reference point being tracked tracked well.

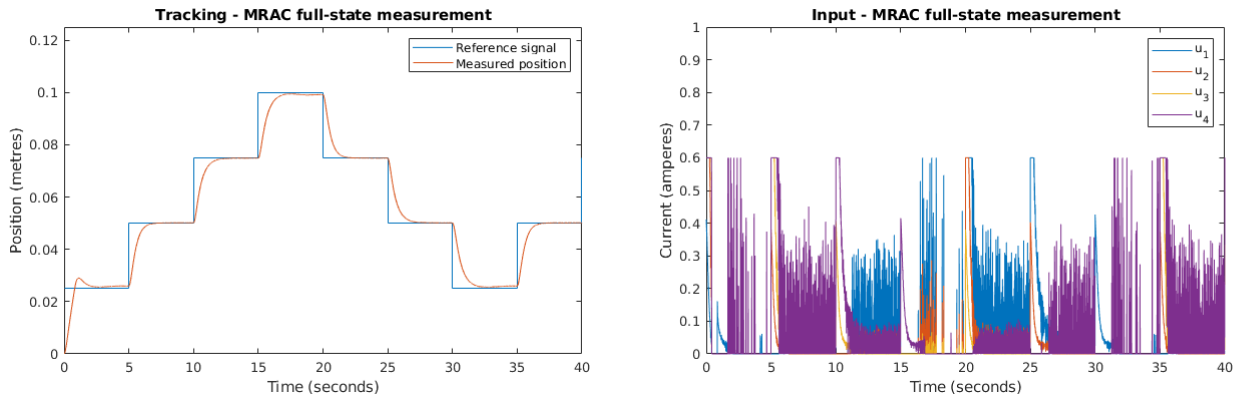


Figure 7: MRAC simulation results - reference tracking (left) and control input (right).

6.3 Experimental results

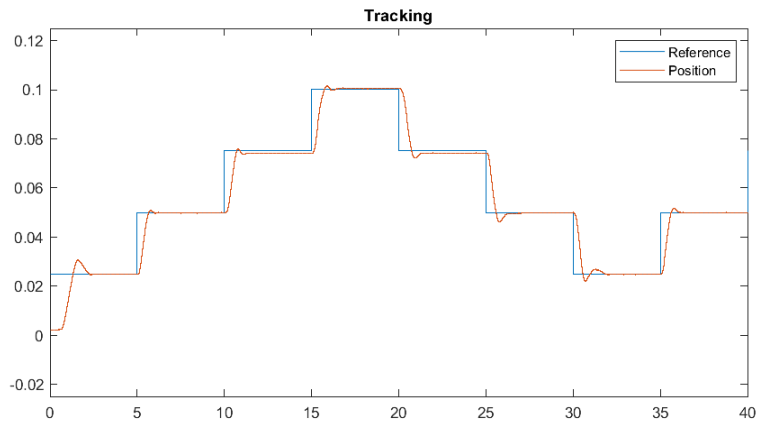


Figure 8: Controller tracking for reference points at the center of the coil

For a staircase set of reference points at the center of the coil, the controller was able to track the required trajectory as shown in figure 8.

The performance of the controller for the reference points set to random points on the coil is given below in the figure 9.

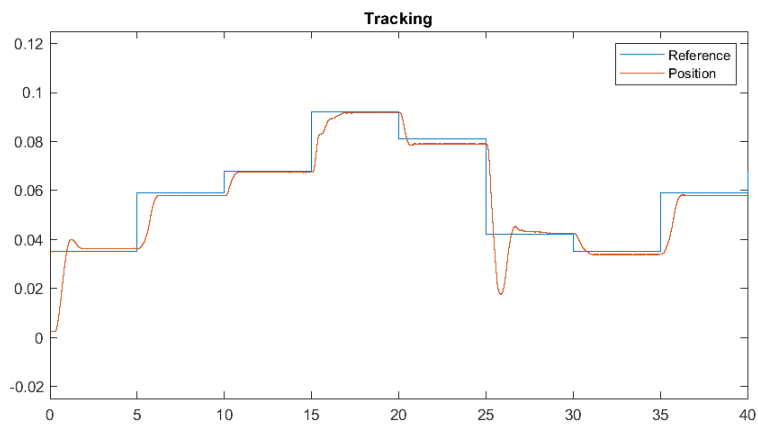


Figure 9: Controller tracking for reference signal at the non-center points of the coil

7 Gain scheduling controller

Gain scheduling is a control approach for non-linear systems that uses a family of linear controllers, each of which provides satisfactory control for a different operating point of the system. A set of observable variables (called scheduling variables) are used to determine the current operating region of the system (in the case of magman, the measured position) and an appropriate linear controller is then enabled. In this section an implementation of gain scheduling with linear controller design using pole-placement for the magman model is discussed and the experimental results are furnished.

7.1 Linearization of the system dynamics around an operating point

The system dynamics are represented using the non-linear model expressed in equation 2.3. Let the state variables be $z_1 = x$ and $z_2 = \dot{x}$, and for the following section consider the j -th input variable i_j^2 to be u_j . Then the system description is

$$\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} z_2 \\ -\frac{b}{m}z_2 + \sum_{j=1}^4 \frac{g_j(z_1)}{m} u_j \end{bmatrix} = f(z_1, z_2, u_1, \dots, u_4) \quad (7.1)$$

One can observe that the above system is at equilibrium for $z_1 \in \mathbb{R}$, $z_2 = 0$ and $u_j = 0$ for $j = 1 \dots 4$. Note that the equilibrium inputs are taken as zero under the assumption that the ramp has no inclination. If it were inclined, then the currents will not be zero. The state matrices A and B for the linearized model description are computed as follows

$$A = \left. \frac{\partial f}{\partial z} \right|_{(z_o, u_o)} \quad \text{and} \quad B = \left. \frac{\partial f}{\partial u} \right|_{(z_o, u_o)}$$

(z_o, u_o) is the point of equilibrium. The scheduling variable is z_1 .

$$A = \left. \frac{\partial f}{\partial z} \right|_{(z_o, u_o)} = \begin{bmatrix} \frac{\partial f_1}{\partial z_1} & \frac{\partial f_1}{\partial z_2} \\ \frac{\partial f_2}{\partial z_1} & \frac{\partial f_2}{\partial z_2} \end{bmatrix}_{(z_o, u_o)} = \begin{bmatrix} 0 & 1 \\ \sum_{j=1}^4 \frac{\partial}{\partial z_1} \frac{g_j(z_1)}{m} u_j & -\frac{b}{m} \end{bmatrix}_{(z_o, u_o)} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b}{m} \end{bmatrix}$$

$$B = \left. \frac{\partial f}{\partial u} \right|_{(z_o, u_o)} = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \frac{\partial f_1}{\partial u_3} & \frac{\partial f_1}{\partial u_4} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \frac{\partial f_2}{\partial u_3} & \frac{\partial f_2}{\partial u_4} \end{bmatrix}_{(z_o, u_o)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{g_1(z_{1o})}{m} & \frac{g_2(z_{1o})}{m} & \frac{g_3(z_{1o})}{m} & \frac{g_4(z_{1o})}{m} \end{bmatrix}$$

Therefore, around the o -th operating point x_o the linear system description is

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & -\frac{b}{m} \end{bmatrix}}_{A_o} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{g_1(x_o)}{m} & \frac{g_2(x_o)}{m} & \frac{g_3(x_o)}{m} & \frac{g_4(x_o)}{m} \end{bmatrix}}_{B_o} \begin{bmatrix} i_1^2 \\ i_2^2 \\ i_3^2 \\ i_4^2 \end{bmatrix} \quad (7.2)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

The system has been linearized and tested for different number of operating points which are evenly distributed in the range $[0, 0.125]$ m. Although the coils are placed at positions that are multiples of 0.025m (up to the fourth multiple) which are within the manipulating region, the system has to be linearized over the entire range over which the ball can move. If the ball is at the position 0 m (before the first coil) or at the position 0.125 m (after the fourth coil) and the system is not linearized at these points then the ball cannot be pulled back to the region within which it's position can be manipulated.

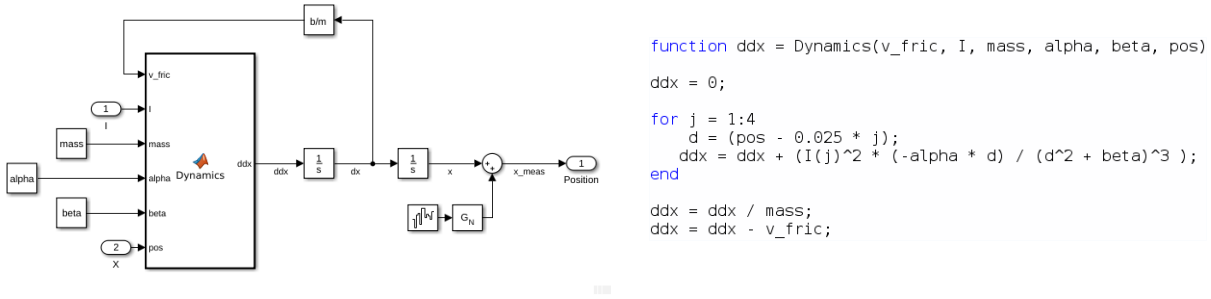


Figure 10: The model used in simulink on MATLAB and the code for the *Dynamics* function block.

7.2 Controller design for the linearized plants and controller scheduling

For all the linearized plants at the choice of operating points, the state-feedback gain (K) has been computed using pole-placement and have been stored in a look-up table (a vector of matrices in MATLAB). For the above model, $K \in \mathbb{R}^{4 \times 2}$. The feedback gain for the o -th operating point is computed using the MATLAB place command: `>> K(o) = place(A(o), B(o), poles);`

The controller is selected in real-time using a very simple scheduling algorithm. The index of the gain in the look-up table is computed based on the operating point that is closest to the current position. In MATLAB, `[~, index] = min(abs(pos - OR));`, where OR is the vector of operating points.

7.3 Simulation results for gain scheduling

In this section, the simulation results for a different number of operating points are furnished. The reference signal used is a repeating stair for the coil centers and the poles are placed at -4 and -5 .

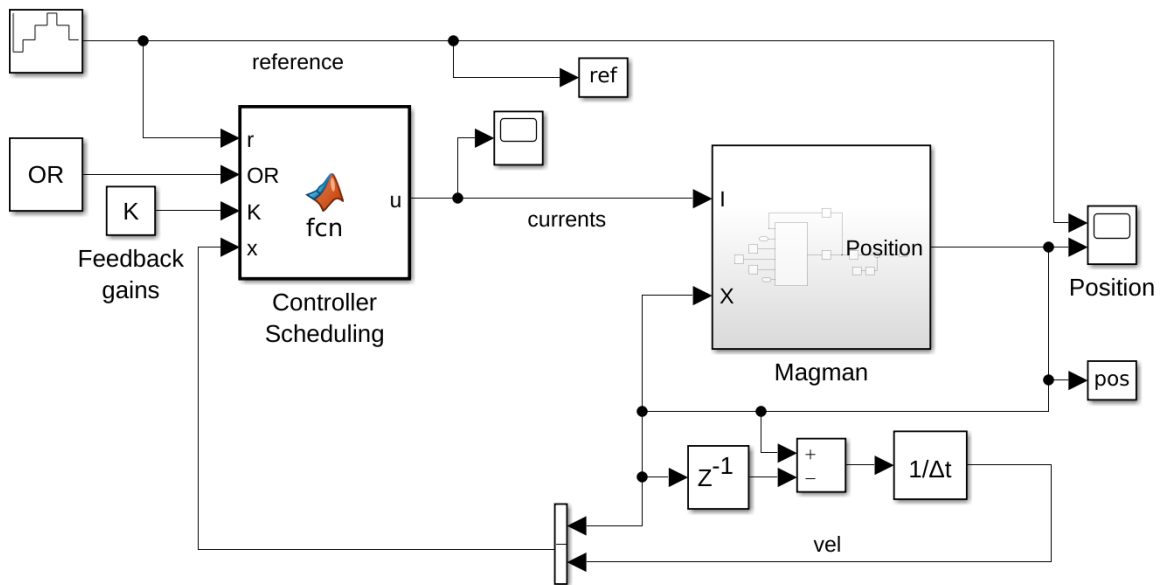


Figure 11: The controller with the look-up table (K) fed to the scheduling algorithm

The choice of the number of linear controllers depends on the system requirements and also on the availability of memory on the system in a real-time setting.

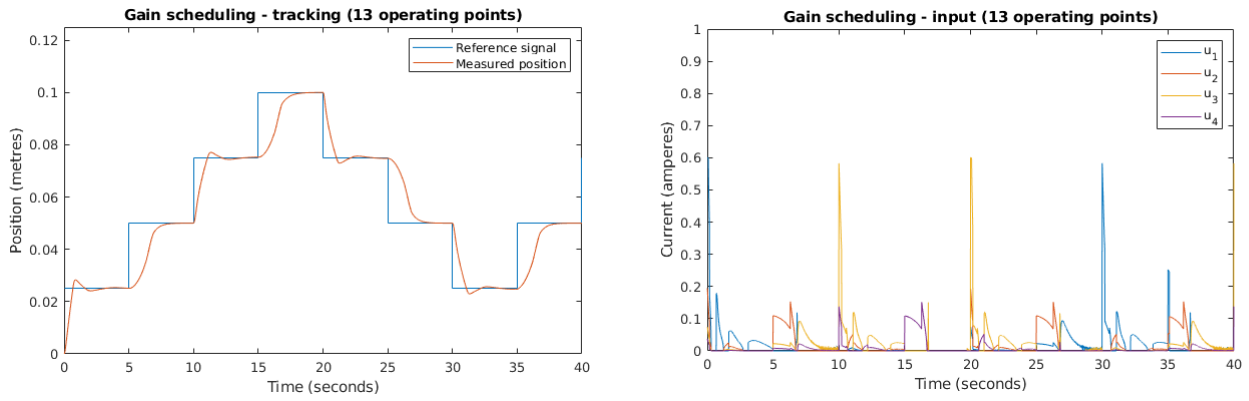


Figure 12: Simulation results for 13 operating points.

It is clearly evident that the performance is not good in the case where the linearization is done at only 13 points. The settling time, offshoot and rise time for each step show no discernible pattern.

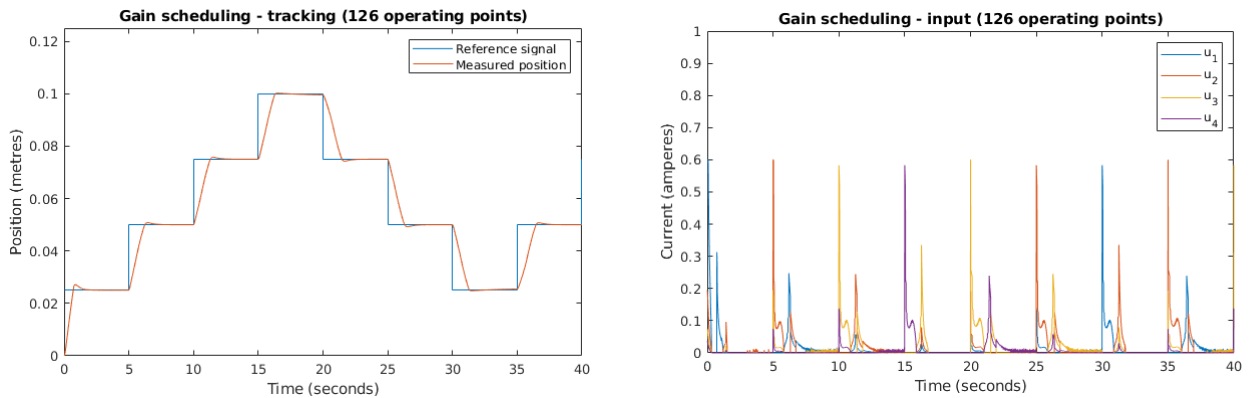


Figure 13: Simulation results for 126 operating points.

For the case with 126 operating points, there is a significant improvement in system performance.

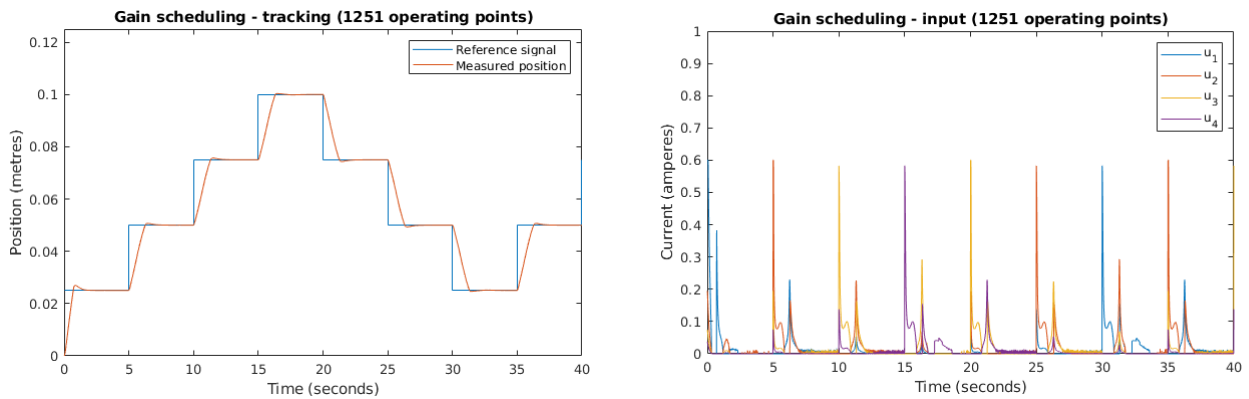


Figure 14: Simulation results for 1251 operating points.

There seems to be no change in performance in the case with 1251 operating points when compared to using 126 operating points. For the current control objective, considering that the speed of the system is limited, 126 linear controllers will suffice. Note that these conclusions are solely for the simulation results and the performance of the controller on the actual system may vary.

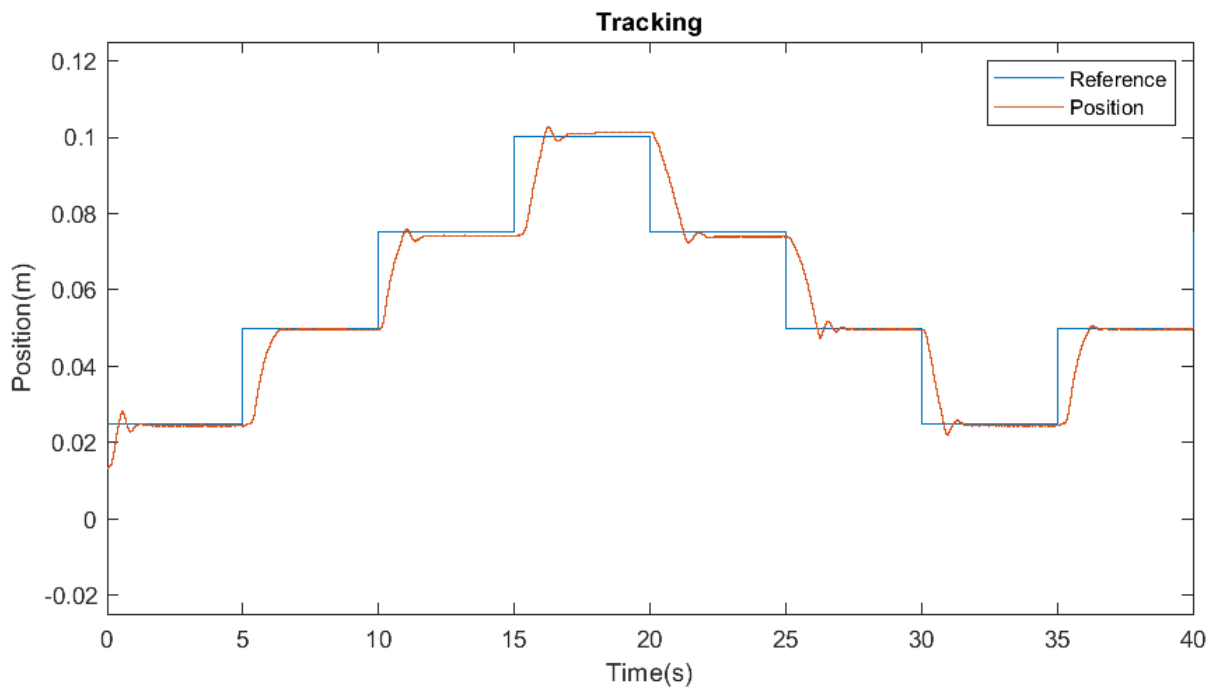


Figure 15: Tracking of the reference signal at the center of the coil by a Gain Scheduling Controller

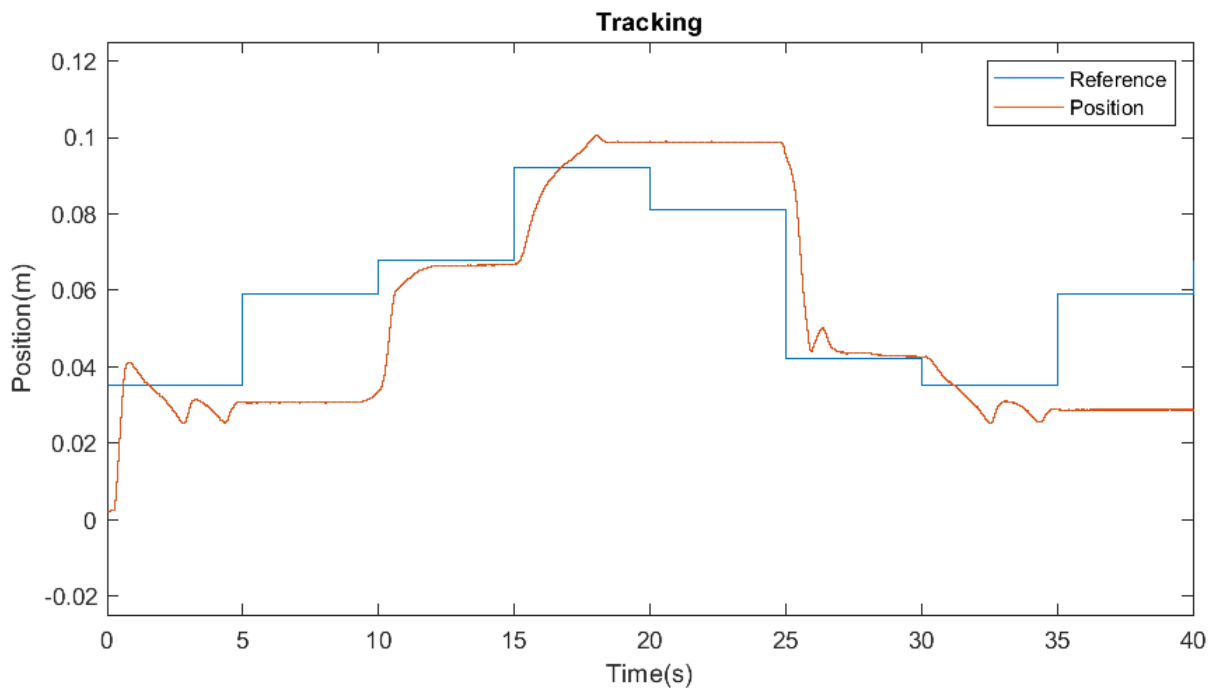


Figure 16: Tracking of the reference signal at random points on the coil by a Gain Scheduling Controller

8 Results

8.1 Pole Placement

Step responses for various reference points on the center of the coil are given below:

Table 1: Step response of pole placement control

Reference Point (m)	0.025	0.05	0.075	0.1	Average
Overshoot	14.1005	9.3654	15.1846	21.4783	16.085
Settling Time	2.7874	1.8854	4.0415	2.3203	2.758
Rise Time	0.5817	0.4578	0.4200	0.4060	0.466

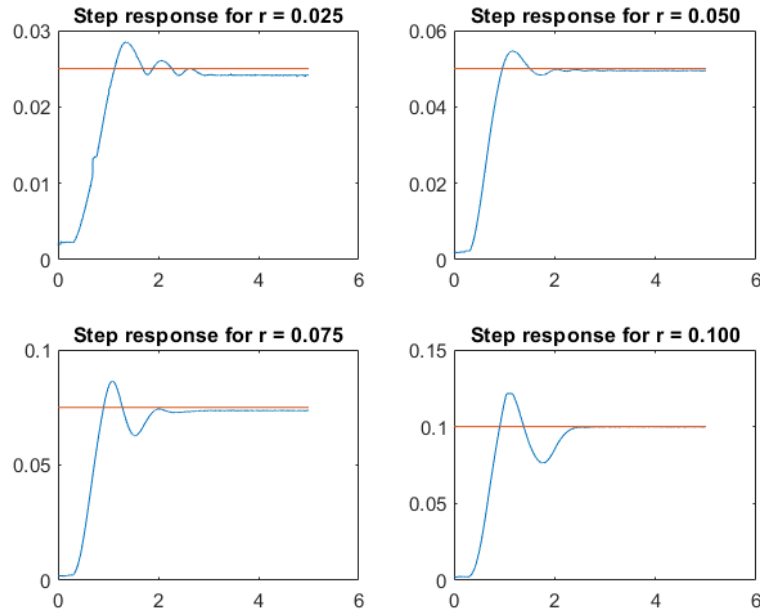


Figure 17: Step response for different reference points in pole placement

8.2 MRAC by full-state measurement

MRAC performed relatively better than the pole placement technique and the results are summarized in the table given below:

Table 2: Step response of the MRAC by full-state measurement

Reference Point (m)	0.025	0.05	0.075	0.1	Average
Overshoot	21.8853	15.5526	19.0964	21.4783	19.5032
Settling Time	9.9964	1.6603	2.0666	2.1209	3.961
Rise Time	0.5153	0.4175	0.4050	0.3959	0.4334

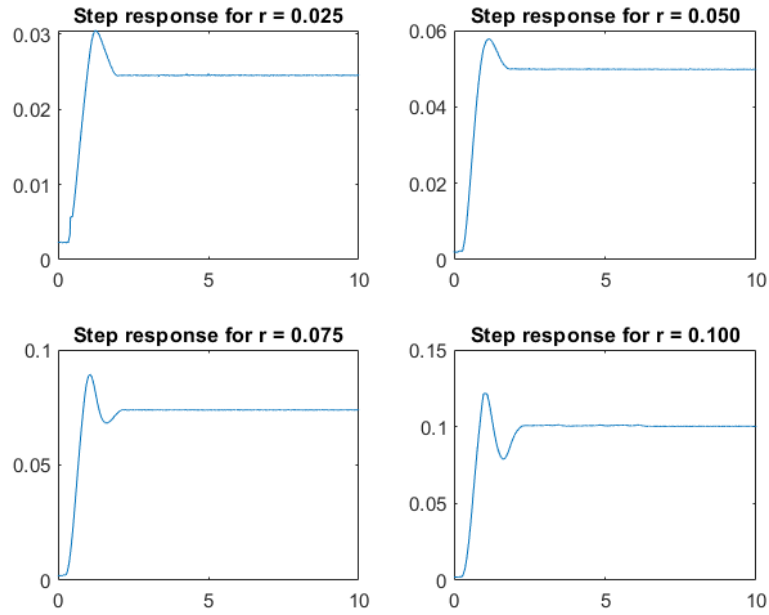


Figure 18: Step response for different reference points in MRAC

8.3 Gain Scheduling

The gain scheduling controller did not perform as well as the other pole placement techniques as can be seen from the figure 19. It might be probably because of high sampling rate which does not let the controller shift to an appropriate gain in time. A lower sampling rate did not work as intended.

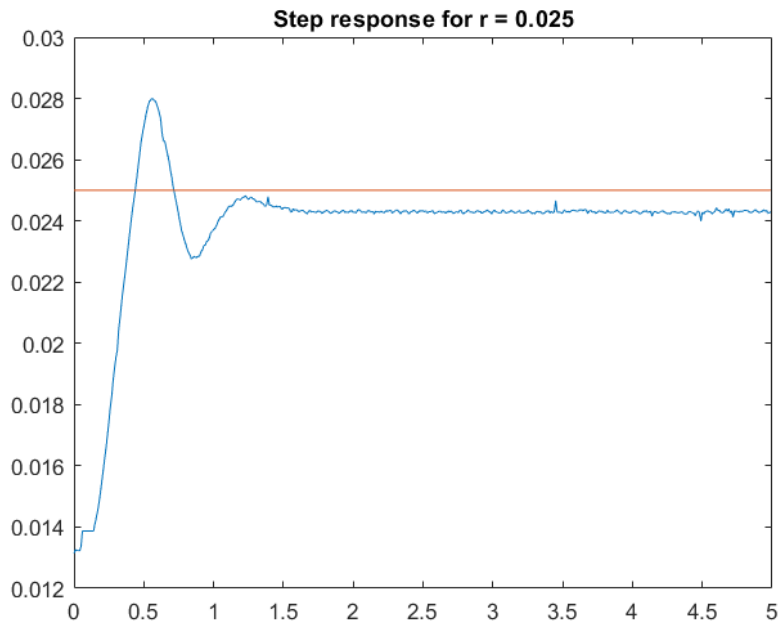


Figure 19: Step response for $r = 0.025$ for the Gain Scheduling Controller

9 Conclusions

Out of the three control approaches implemented and discussed in this report, pole placement and MRAC by full-state measurement performed better than gain scheduling.

Although the step responses showed a better performance for pole placement, MRAC performed better under some conditions. It is possible that the coils behave differently on long term usage due to heating or other effects under which pole placement sometimes fails to give the desired control response.